Physics 3054

Problem Set 11
Due at the Beginning of Lecture on December 12 (Thursday)

1). For an elliptic orbit with a central force

$$\vec{F}(r) = \frac{\alpha}{r^2} \hat{r}$$

show that the time average of the kinetic energy ($T$) and the time average of the potential energy ($U$) satisfy

$$< T > = -\frac{1}{2} < U >$$

where the time average for a quantity $Q$ is defined as

$$< Q > = \frac{1}{\tau} \int_0^\tau Q(t) dt.$$ 

This is known as the Clausius Virial theorem.

2). The two atoms in a diatomic molecule (masses $m_1$ and $m_2$) interact through a potential energy

$$U(r) = \frac{a^2}{4r^4} - \frac{b^2}{3r^3}$$

where $r$ is the separation of the atoms.

(a). Find the equilibrium separation of the atoms and the frequency of small oscillations about the equilibrium assuming that the molecule does not rotate. How much energy must be supplied to this molecule in order to break it up?

(b). Determine the maximum angular momentum which the molecule can have without breaking up, assuming that the motion is in circular orbits. Find the particle separation at the break up angular momentum.

(c). Calculate the velocity of each particle in the laboratory system at break up, assuming that the center of mass is at rest.

3). A point particle of mass $m$ is located at the point $(x_0, y_0, 0)$.

(a). Calculate the tensor of inertia (I) about the origin.

(b). Find the principal axes and interpret the result.
4). A pendulum consists of two masses \((m_a\) and \(m_b\)) connected by a very light rigid rod with a negligible mass as shown in the figure. The pendulum is free to oscillate in the vertical plane about a horizontal axis located at a distance \(a\) from \(m_a\) and at a distance \(b\) from \(m_b\).

(a). Find the location of the center of mass and calculate the moment of inertia of the system about the point \(O\).

(b). Find the Lagrangian of this system and set up the equation of motion for \(\theta\).

(c). Take \(b m_b > a m_a\) and determine the frequency of oscillation for small angles of displacement from the vertical axis.

(d). Derive an exact expression in the form of an integral for the period of the pendulum \((|\theta_{\text{MAX}}| < \pi\)).

(e). Find the minimum angular velocity which must be given to the system starting at equilibrium if it is to continue in rotation instead of oscillating.