PHYS 3053: Physical Mechanics II
Problem Set 2 — Due September 6

1) Show that $e^{i\theta} = \cos \theta + i \sin \theta$, $i = \sqrt{-1}$. (This is Euler’s formula.)

HINT: Apply the Taylor series of every function.

2) An underdamped harmonic oscillator has the equation of motion of the form

$$\left(\frac{d^2}{dt^2} + 2\gamma \frac{d}{dt} + \omega_0^2\right)x(t) = 0$$

where $\omega_0 = \sqrt{k/m}$, $k =$ the spring constant, $m =$ mass, and $\gamma < \omega_0$. The solution of this equation can be expressed as $A e^{-\gamma t} \cos(\omega t + \phi)$ with $\omega = \sqrt{\omega_0^2 - \gamma^2}$. Derive expressions for $A$ and $\phi$ with the initial conditions $x_0 = x(0)$ and $v_0 = v(0)$ at $t = 0$.

3) (a) An undamped electric circuit is described by the equation

$$L \frac{d^2Q}{dt^2} + \frac{Q}{C} = 0,$$

where $Q =$ charge, $L =$ inductance, and $C =$ capacitance. What are the angular frequency, the frequency, and the period of this oscillator?

(b) Now suppose we have the same circuit except we introduce a resistor with resistance $R$. In the underdamped case, the circuit still exhibits oscillatory behavior. What is the new angular frequency, the frequency, and the period? Is the oscillation faster or slower?

(c) As the resistance $R$ is increased, the period of oscillation increases, eventually going to infinity. What is the value of $R$ for which the period becomes infinite, and to what kind of damping does this situation correspond?

4) When an object of mass $m$ moves through a gas, there is a frictional force proportional to $v$ such that $F(v) = -mbv$, where $m$ and $b$ are constants. Derive an expression for $x(t)$ of this object with the initial conditions $v_0 = v(t_0)$ and $x_0 = x(t_0)$ at $t = t_0$ in two ways:

(a) by applying separation of variables, and

(b) by finding the general solution

$$x(t) = \sum_{i=1}^{2} c_i e^{\lambda_i t}$$

of a second order linear homogeneous differential equation that has the characteristic equation in terms of $\lambda$ with roots $\lambda_i$. 